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A modified shear deformation theory associated with the four-node quadrilateral element for bending and free vibration analyses of plates

Lan Hoang That Ton[1]
[1] Dept. of Civil Engineering at HCMC University of Architecture, HCMC, Vietnam.

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ABSTRACT

Few years ago, the higher-order shear deformation theory of Shi was proven effective in the analysis of structures. In this paper, the author introduces a modification of above theory combined with four-node quadrilateral elements for bending and free vibration analyses of plates. Shear correction factor is not required here. The acceptable results obtained from the proposed element are a necessary prerequisite for the development of other problems in the near future.

Corresponding Author:

Lan Hoang That Ton.
Department of Civil Engineering,
HCMC University of Architecture, VN
Email: tltechonlinesom@gmail.com

1. INTRODUCTION

A slew of plate theories have been introduced in the past few decades [1-8]. For instance, a higher-order shear deformation theory was used to analyse laminated anisotropic composite plates for deflections, stresses, natural frequencies and buckling loads by authors Phan and Reddy [8]. This theory presented parabolic distribution of the transverse shear stresses and without any shear correction factors. An other shear deformation theory was introduced in [9] to study free vibration analysis of the simply supported functionally graded plates. The shear functions gave in this paper satisfy the stress-free boundary conditions on the top and bottom surfaces and vary with a gradient index of functionally graded plates without using any shear correction factor. Besides, the displacement field was expressed as undetermined integral terms. The governing differential equation and boundary conditions were derived based on Hamilton's principle. In other way, displacement based finite element using equivalent-single-layer theory was proposed by Rajaneesh *et al.* [10]. With some benefits of a new first-order shear deformation theory, they were applied to compute stiffness, mass and force matrices and based on only two variables against three variables in Mindlin's first order theory, etc. The third-order shear deformation plate theories have some advantages related to the quadratic variation of the transverse shear stresses and strains along the thickness as well as the shear locking free. The author Shi [6] introduced another third-order shear deformation plate theory based on meticulous kinematics of displacements, initially applied to static analysis of isotropic and orthotropic beams and plates. The solutions obtained by Shi's theory have more reliable than others. Beside the analytical approaches, the numerical methods are used in the structural analyses [11-20]. In this study, the bending and free vibration analyses of plates based on the finite element procedure and the modification of Shi's theory is introduced as the main objective.

The rest sections are built as follows. The modification of Shi theory and the finite element formulation for bending and free vibration analyses of plates are presented in Sect.2. The results of proposed method are shown and compared with other references in Sect.3. Sect.4 draws out some remarks of this paper.

2. FORMULATIONS

In this paper, the author proposes a modification of Shi theory expressed as follows

$$u(x,y,z)=u_0(x,y)+f(z)\phi_x^b(x,y)+g(z)\phi_x^s(x,y) \quad (1)$$

$$v(x,y,z)=v_0(x,y)+f(z)\phi_y^b(x,y)+g(z)\phi_y^s(x,y) \quad (2)$$

$$w(x,y,z)=w_0(x,y) \quad (3)$$

in which

$$f(z)=\frac{1}{4}z-\frac{5}{3h^2}z^3 \quad \text{and} \quad g(z)=\frac{5}{4}z-\frac{5}{3h^2}z^3 \quad (4)$$

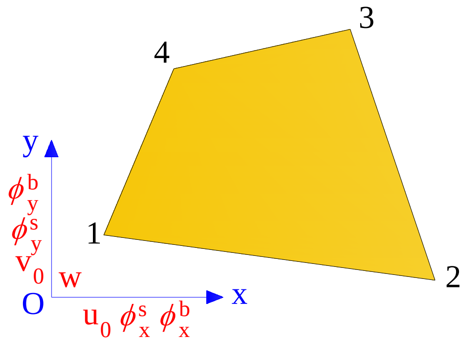

Figure 1. The proposed element with seven degrees of freedom per node

With the small strain assumptions, the strain-displacement relations can be shown

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{Bmatrix}=\begin{Bmatrix} u_{0,x}+\left(5\phi_{x,x}^s+\phi_{x,x}^b\right)z\frac{1}{4}+\left(\phi_{x,x}^s+\phi_{x,x}^b\right)z^3\left(\frac{-5}{3h^2}\right)\\ v_{0,y}+\left(5\phi_{y,y}^s+\phi_{y,y}^b\right)z\frac{1}{4}+\left(\phi_{y,y}^s+\phi_{y,y}^b\right)z^3\left(\frac{-5}{3h^2}\right)\\ u_{0,y}+v_{0,x}+\left(5\phi_{x,y}^s+5\phi_{y,x}^s+\phi_{x,y}^b+\phi_{y,x}^b\right)z\frac{1}{4}+\left(\phi_{x,y}^s+\phi_{y,x}^s+\phi_{x,y}^b+\phi_{y,x}^b\right)z^3\left(\frac{-5}{3h^2}\right)\end{Bmatrix} \quad (5)$$

$$\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=\begin{Bmatrix}\left(\frac{5}{4}\phi_y^s+\frac{1}{4}\phi_y^b+w_{0,y}\right)+\left(\phi_y^s+\phi_y^b\right)z^2\left(\frac{-5}{h^2}\right)\\ \left(\frac{5}{4}\phi_x^s+\frac{1}{4}\phi_x^b+w_{0,x}\right)+\left(\phi_x^s+\phi_x^b\right)z^2\left(\frac{-5}{h^2}\right)\end{Bmatrix} \quad (6)$$

By setting

$$\boldsymbol{\varepsilon}^{(0)}=\begin{Bmatrix}u_{0,x}\\ v_{0,y}\\ u_{0,y}+v_{0,x}\end{Bmatrix};\ \boldsymbol{\varepsilon}^{(1)}=\begin{Bmatrix}\left(\frac{5}{4}\phi_{x,x}^s+\frac{1}{4}\phi_{x,x}^b\right)\\ \left(\frac{5}{4}\phi_{y,y}^s+\frac{1}{4}\phi_{y,y}^b\right)\\ \left(\frac{5}{4}\phi_{x,y}^s+\frac{5}{4}\phi_{y,x}^s+\frac{1}{4}\phi_{x,y}^b+\frac{1}{4}\phi_{y,x}^b\right)\end{Bmatrix};\ \boldsymbol{\varepsilon}^{(3)}=\frac{-5}{3h^2}\begin{Bmatrix}\phi_{x,x}^s+\phi_{x,x}^b\\ \phi_{y,y}^s+\phi_{y,y}^b\\ \phi_{x,y}^s+\phi_{x,y}^b+\phi_{y,x}^s+\phi_{y,x}^b\end{Bmatrix} \quad (7)$$

$$\boldsymbol{\gamma}^{(0)}=\begin{Bmatrix}\frac{5}{4}\phi_y^s+\frac{1}{4}\phi_y^b+w_{0,y}\\ \frac{5}{4}\phi_x^s+\frac{1}{4}\phi_x^b+w_{0,x}\end{Bmatrix};\ \boldsymbol{\gamma}^{(2)}=\frac{-5}{h^2}\begin{Bmatrix}\phi_y^s+\phi_y^b\\ \phi_x^s+\phi_x^b\end{Bmatrix} \quad (8)$$

Eqs. (5) and (6) become

$$\boldsymbol{\varepsilon}=\boldsymbol{\varepsilon}^{(0)}+z\boldsymbol{\varepsilon}^{(1)}+z^3\boldsymbol{\varepsilon}^{(3)} \quad (9)$$

$$\boldsymbol{\gamma}=\boldsymbol{\gamma}^{(0)}+z^2\boldsymbol{\gamma}^{(2)} \quad (10)$$

Under Hooke's law, the constitutive equations are given

$$\boldsymbol{\sigma} = \mathbf{D}_m(\boldsymbol{\varepsilon}^{(0)} + \mathbf{z}\boldsymbol{\varepsilon}^{(1)} + \mathbf{z}^3\boldsymbol{\varepsilon}^{(3)}) \tag{11}$$

$$\boldsymbol{\tau} = \mathbf{D}_s\left(\boldsymbol{\gamma}^{(0)} + \mathbf{z}^2\boldsymbol{\gamma}^{(2)}\right) \tag{12}$$

in which

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \sigma_y & \sigma_{xy} \end{bmatrix}^{\mathrm{T}} \tag{13}$$

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_{yz} & \tau_{xz} \end{bmatrix}^{\mathrm{T}} \tag{14}$$

$$\mathbf{D}_m = \frac{E}{1-v^2}\begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & (1-v)/2 \end{bmatrix} \tag{15}$$

$$\mathbf{D}_s = \frac{E}{2(1+v)}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{16}$$

The displacements can hence be approximated as

$$\mathbf{u} = \mathbf{N}\mathbf{q}_e \tag{17}$$

with

$$\mathbf{u} = \begin{bmatrix} u_0 & v_0 & w_0 & \phi_x^s & \phi_y^s & \phi_x^b & \phi_y^b \end{bmatrix}^{\mathrm{T}} \tag{18}$$

$$\mathbf{N} = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \tag{19}$$

$$\mathbf{q}_e = \begin{bmatrix} \mathbf{q}_{1e} & \mathbf{q}_{2e} & \mathbf{q}_{3e} & \mathbf{q}_{4e} \end{bmatrix}^{\mathrm{T}} \tag{20}$$

$\mathbf{q}_e$ and $\mathbf{N}$ are the unknown displacement vector and the shape function vector. From above equations, the strain can be rewritten

$$\boldsymbol{\varepsilon} = \mathbf{B}1\mathbf{q}_e + \mathbf{B}2\mathbf{q}_e + \mathbf{B}3\mathbf{q}_e \tag{21}$$

$$\boldsymbol{\gamma} = \mathbf{B}4\mathbf{q}_e + \mathbf{B}5\mathbf{q}_e \tag{22}$$

in which

$$\mathbf{B}1 = \sum_{i=1}^{4}\begin{bmatrix} \mathbf{N}_{i,x} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{N}_{i,y} & 0 & 0 & 0 & 0 & 0 \\ \mathbf{N}_{i,y} & \mathbf{N}_{i,x} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{23}$$

$$\mathbf{B}2 = \sum_{i=1}^{4}\begin{bmatrix} 0 & 0 & 0 & \frac{5}{4}\mathbf{N}_{i,x} & 0 & \frac{1}{4}\mathbf{N}_{i,x} & 0 \\ 0 & 0 & 0 & 0 & \frac{5}{4}\mathbf{N}_{i,y} & 0 & \frac{1}{4}\mathbf{N}_{i,y} \\ 0 & 0 & 0 & \frac{5}{4}\mathbf{N}_{i,y} & \frac{5}{4}\mathbf{N}_{i,x} & \frac{1}{4}\mathbf{N}_{i,y} & \frac{1}{4}\mathbf{N}_{i,x} \end{bmatrix} \tag{24}$$

$$\mathbf{B}3 = -\frac{5}{3h^2}\sum_{i=1}^{4}\begin{bmatrix} 0 & 0 & 0 & \mathbf{N}_{i,x} & 0 & \mathbf{N}_{i,x} & 0 \\ 0 & 0 & 0 & 0 & \mathbf{N}_{i,y} & 0 & \mathbf{N}_{i,y} \\ 0 & 0 & 0 & \mathbf{N}_{i,y} & \mathbf{N}_{i,x} & \mathbf{N}_{i,y} & \mathbf{N}_{i,x} \end{bmatrix} \tag{25}$$

$$\mathbf{B}4 = \sum_{i=1}^{4}\begin{bmatrix} 0 & 0 & \mathbf{N}_{i,y} & 0 & \frac{5}{4}\mathbf{N}_i & 0 & \frac{1}{4}\mathbf{N}_i \\ 0 & 0 & \mathbf{N}_{i,x} & \frac{5}{4}\mathbf{N}_i & 0 & \frac{1}{4}\mathbf{N}_i & 0 \end{bmatrix} \tag{26}$$

$$\mathbf{B}5 = -\frac{5}{h^2}\sum_{i=1}^{4}\begin{bmatrix} 0 & 0 & 0 & 0 & \mathbf{N}_i & 0 & \mathbf{N}_i \\ 0 & 0 & 0 & \mathbf{N}_i & 0 & \mathbf{N}_i & 0 \end{bmatrix} \tag{27}$$

The total strain energy of a plate can be given by

$$U=\frac{1}{2}\int_{V_e}\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{\sigma}dV-\int_{S_e}\mathbf{u}^{\mathrm{T}}\mathbf{f}dS=\frac{1}{2}\mathbf{q}_e^{\mathrm{T}}\int_{S_e}\begin{pmatrix}\mathbf{B1}^{\mathrm{T}}\mathbf{AB1}+\mathbf{B1}^{\mathrm{T}}\mathbf{BB2}+\mathbf{B1}^{\mathrm{T}}\mathbf{EB3}+\\+\mathbf{B2}^{\mathrm{T}}\mathbf{BB1}+\mathbf{B2}^{\mathrm{T}}\mathbf{DB2}+\mathbf{B2}^{\mathrm{T}}\mathbf{FB3}+\\+\mathbf{B3}^{\mathrm{T}}\mathbf{EB1}+\mathbf{B3}^{\mathrm{T}}\mathbf{FB2}+\mathbf{B3}^{\mathrm{T}}\mathbf{HB3}+\\+\mathbf{B4}^{\mathrm{T}}\hat{\mathbf{A}}\mathbf{B4}+\mathbf{B4}^{\mathrm{T}}\hat{\mathbf{B}}\mathbf{B5}+\mathbf{B5}^{\mathrm{T}}\hat{\mathbf{B}}\mathbf{B4}+\\+\mathbf{B5}^{\mathrm{T}}\hat{\mathbf{D}}\mathbf{B5}\end{pmatrix}dS\mathbf{q}_e-\mathbf{q}_e^{\mathrm{T}}\int_{S_e}\mathbf{N}^{\mathrm{T}}\mathbf{f}dS= \quad (28)$$

$$=\frac{1}{2}\mathbf{q}_e^{\mathrm{T}}\mathbf{K}_e\mathbf{q}_e-\mathbf{q}_e^{\mathrm{T}}\mathbf{F}_e=\mathbf{q}_e^{\mathrm{T}}\left(\frac{1}{2}\mathbf{K}_e\mathbf{q}_e-\mathbf{F}_e\right)$$

with

$$\left(\mathbf{A},\ \mathbf{B},\ \mathbf{D},\ \mathbf{E},\ \mathbf{F},\ \mathbf{H}\right)=\int_{-h/2}^{h/2}\left(1,\ z,\ z^2,\ z^3,\ z^4,\ z^6\right)\mathbf{D}_m dz \quad (29)$$

$$\left(\hat{\mathbf{A}},\ \hat{\mathbf{B}},\ \hat{\mathbf{D}}\right)=\int_{-h/2}^{h/2}\left(1,\ z^2,\ z^4\right)\mathbf{D}_s dz \quad (30)$$

The kinetic energy is shown

$$T=\frac{1}{2}\int_{V_e}\dot{\mathbf{u}}^{\mathrm{T}}\boldsymbol{\rho}(z)\dot{\mathbf{u}}dV=\frac{1}{2}\dot{\mathbf{q}}_e^{\mathrm{T}}\left\{\int_{V_e}\mathbf{N}^{\mathrm{T}}\mathbf{L}^{\mathrm{T}}\boldsymbol{\rho}(z)\mathbf{L}\mathbf{N}dV\right\}\dot{\mathbf{q}}_e=\frac{1}{2}\dot{\mathbf{q}}_e^{\mathrm{T}}\mathbf{M}_e\dot{\mathbf{q}}_e \quad (31)$$

in which $\mathbf{L}$ is clearly decribed as

$$\mathbf{L}=\begin{bmatrix}1&0&0&g(z)&0&f(z)&0\\0&1&0&0&g(z)&0&f(z)\\0&0&1&0&0&0&0\end{bmatrix} \quad (32)$$

and the mass matrix of element is presented

$$\mathbf{M}_e=\int_{V_e}\mathbf{N}^{\mathrm{T}}\mathbf{L}^{\mathrm{T}}\boldsymbol{\rho}(z)\mathbf{L}\mathbf{N}dV=\int_{S_e}\mathbf{N}^{\mathrm{T}}\left(\int_{-h/2}^{h/2}\boldsymbol{\rho}(z)\mathbf{L}^{\mathrm{T}}\mathbf{L}dz\right)\mathbf{N}dS \quad (33)$$

For bending analysis

$$\mathbf{Kd}=\mathbf{F} \quad (34)$$

For free vibration analysis

$$\left(\mathbf{K}-\omega^2\mathbf{M}\right)\mathbf{d}=0 \quad (35)$$

3. RESULTS

In this section, the author will present some examples to illustrate the applicability of the proposed element for bending and free vibration analyses.

Firstly, the simply supported plate with side lengths a, b and thickness h is studied. The uniform load q is considered to be illustrated in Figure 2. The normalized deflection and stress at central point can be depicted as $\bar{w}=\frac{Eh^3}{qa^4}w\left(\frac{a}{2},\frac{b}{2}\right)$ and $\bar{\sigma}_x=\frac{h^2}{qa^2}\sigma_x\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right)$.

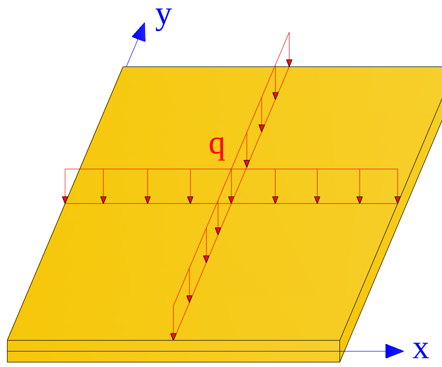

Figure 2. The plate with uniform load

By changing the ratios *b/a* and *a/h*, the results of this paper are shown in Table 1 and compared with solutions of author Reddy [5] based on a higher-order shear deformation theory. A good agreement is observed for all values of above ratios confirming the accuracy of present results.

Secondly, to verify the accuracy of present results related to the modified shear deformation theory without any shear correction factors, the nondimensional deflections $\tilde{\bar{w}} = \frac{100D}{qa^4} w\left(\frac{a}{2}, \frac{b}{2}\right)$ are calculated and compared in Table 2 with exact solutions [7] for various values of length to thickness ratio *a/h* from thick to thin plate. It can be seen that the proposed method is not only accurate but also efficient in static bending analysis.

Table 1. The normalized central deflection and stress of simply supported plates

b/a	*a/h*	Theory	$\bar{w}$	$\bar{\sigma}_x$
1	5	HSDT [5]	0.0535	0.2944
		Present	0.0535	0.2942
	10	HSDT [5]	0.0467	0.2890
		Present	0.0466	0.2887
	100	HSDT [5]	0.0444	0.2873
		Present	0.0443	0.2872
2	5	HSDT [5]	0.1248	0.6202
		Present	0.1247	0.6200
	10	HSDT [5]	0.1142	0.6125
		Present	0.1141	0.6123
	100	HSDT [5]	0.1106	0.6100
		Present	0.1105	0.6098

Table 2. The nondimensional deflections for thick and thin plates

b/a	*a/h*	Theory	$\tilde{\bar{w}}$
2	5	Exact [7]	1.1428
		Present	1.1423
	10	Exact [7]	1.0454
		Present	1.0445
	25	Exact [7]	1.0181
		Present	1.0171
	1000	Exact [7]	1.0129
		Present	1.0119

Finally, the natural frequencies of a simply supported plate are calculated. These parameters can be normalized by formulating $\bar{\omega} = \omega a^2 \sqrt{\rho h / D}$. Table 3 shows the first three normalized frequencies of this paper and other results based on a higher-order shear deformation theory of author Reddy [5]. Again, the difference between the two approaches is negligible.

Table 3. The normalized frequencies of rectangular plate

a/b	*h/a*	Theory	Mode 1	Mode 2	Mode 3
0.5	0.01	HSDT [5]	12.3342	19.7320	32.0572
		Present	12.4344	20.1185	33.8274
	0.1	HSDT [5]	12.0675	19.0653	30.3623
		Present	12.1572	19.3968	31.8088
	0.2	HSDT [5]	11.3717	17.4523	26.6838
		Present	11.3929	17.5016	26.8765
1	0.01	HSDT [5]	19.7320	49.3032	49.3032
		Present	19.8855	50.7044	50.7044
	0.1	HSDT [5]	19.0653	45.4869	45.4869
		Present	19.1872	46.3754	46.3754
	0.2	HSDT [5]	17.4523	38.1883	38.1883

	Present	17.3483	36.4226	36.4226

Moreover, the first six mode shapes of this structure can be also seen as below:

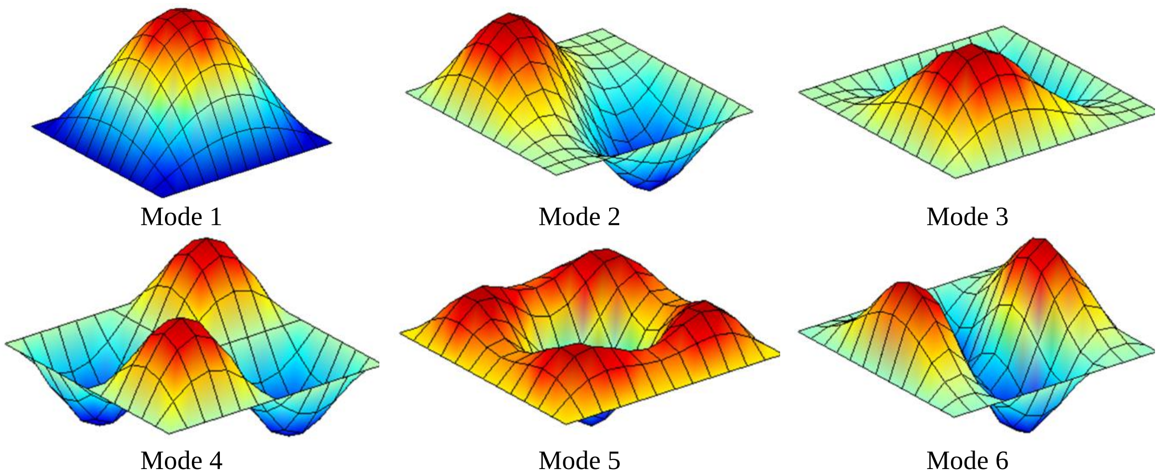

Figure 3. The first six mode shapes of simply supported plate with *a/b* = 1 and *h/a* = 0.01.

4. CONCLUSION

This paper deals with the bending and free vibration analyses of plates related to finite element procedure under a modified shear deformation theory. The computational solutions of the proposed finite element are depicted and compared with the results in other literatures. The effects of some parameters like aspect ratio, thickness ratio are also shown in above section. It can be concluded that the present method is not only accurate but also efficient. The paper also helps to supplement the knowledge for new researchers.

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